REPORT
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Cooperator	Jet Propulsion Laboratory California Institute of Technology 4800 Oak Grove Drive Pasadena, California
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Investigation of	Surface Electromagnetic Reflection Characteristics
Subject of Report	Back-Scattering of Circularly Polarized Waves from Slightly Rough Surfaces
Submitted by	J. L. George Antenna Laboratory Department of Electrical Engineering
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BACK - SCATTERING OF CIRCULARLY POLARIZED WAVES FROM SLIGHTLY ROUGH SURFACES

I. INTRODUCTION

The analysis of the interaction of electromagnetic waves with a slightly rough surface has previously been carried out[1] for the case of horizontal and vertical polarization. The purpose of this paper is to extend these results to the case of circular polarization, and to attempt to determine the radar back-scattering coefficient of the Moon by this method.

When a linearly polarized wave is scattered by a slightly rough surface, the scattered wave in any fixed direction is composed of two orthogonal components. That is, the scattered wave for a vertically polarized incident wave will contain not only a vertically polarized component, but also a horizontally polarized component, the magnitudes of these terms depending upon the direction of the scatterings. If the incident wave is circularly polarized the return will contain both right- and left-circular polarization, the magnitudes again being dependent upon the direction of scattering. In the following work a system of double subscripts will be used to identify the polarization state of the different scattering terms. The first subscript indicates the polarization of the incident wave; and the second subscript, the polarization of the scattered wave. The subscripts are:

- h - horizontal
- v - vertical
- r - right-circular
- l - left-circular

The conventions of electrical engineering practice are used. The superscript s indicates a scattered field. Since a circularly polarized field may be thought of as consisting of a horizontal component E_h, and a vertical component E_v of the same amplitude but with a difference in phase of 90°, the previously derived results for scattering of linearly polarized waves[1] may be extended to the circularly polarized case by taking suitable linear combinations. For example, when a circularly polarized incident field is scattered by a slightly rough surface, the scattered field will consist of two horizontal components in phase quadrature, E^s_{hh} and E^s_{vh} , and two vertical components in phase quadrature, E_{hv} and E_{vv} (See Fig. 1). If the incident field is right-circularly polarized, the total horizontal component and vertical component of the scattered wave are, respectively,

(1) $E^s_{rh} = \frac{1}{\sqrt{2}} \left(E^s_{hh} + j\, E^s_{vh} \right)$ $\left\{ \begin{array}{l} \text{horizontal component of scattered} \\ \text{field due to right-circular polarized} \\ \text{incident wave} \end{array} \right\}$

(2) $E^s_{rv} = \frac{1}{\sqrt{2}} \left(E^s_{hv} + j\, E^s_{vv} \right)$ $\left\{ \begin{array}{l} \text{vertical component of scattered field} \\ \text{due to right-circular polarized} \\ \text{incident wave} \end{array} \right\}$

These polarization states may be resolved into a right-circular and a left-circular scattered wave:

(3) $$E^s_{rr} = \frac{1}{\sqrt{2}} \left(E^s_{rh} + j\, E^s_{rv} \right) = \frac{1}{2} \left[E^s_{hh} - E^s_{vv} + j \left(E^s_{vh} + E^s_{hv} \right) \right]$$ $\left\{ \begin{array}{l} \text{right-circular component} \\ \text{of scattered wave due to} \\ \text{right-circular incident} \\ \text{wave} \end{array} \right\}$

(4) $$E^s_{r\ell} = \frac{1}{\sqrt{2}} \left(E^s_{rh} - jE^s_{rv} \right) = \frac{1}{2} \left[E^s_{hh} + E^s_{vv} + j \left(E^s_{vh} - E^s_{hv} \right) \right] .$$ $\left\{ \begin{array}{l} \text{left-circular component} \\ \text{of scattered wave due to} \\ \text{right-circular incident} \\ \text{wave} \end{array} \right\}$

In this paper only the case of back-scattering will be considered. Equation (3) and (4) are thus simplified because the cross-polarized terms E_{hv}, E_{vh}, are zero for back-scattering from the kind of slightly rough surface we wish to consider[1] ;

(5) $$E^{BS}_{rr} = \frac{1}{2} \left(E^s_{hh} - E^s_{vv} \right) \qquad \begin{array}{c} (\theta_o = \theta_s) \\ \phi_s = 180^\circ \end{array}$$

(6) $$E^{BS}_{r\ell} = \frac{1}{2} \left(E^s_{hh} + E^s_{vv} \right) \qquad \begin{array}{c} (\theta_o = \theta_s) \\ \phi_s = 180^\circ \end{array} .$$

If the incident wave possesses left-circular polarization, the results are:

(7) $$E^{BS}_{\ell r} = \frac{1}{2} \left(E^s_{hh} + E^s_{vv} \right)$$

(8) $$E^{BS}_{\ell\ell} = \frac{1}{2} \left(E^s_{hh} - E^s_{vv} \right) .$$

The scattered fields E^s_{hh} and E^s_{vv} for a slightly rough surface may be found in Ref. 1. It is shown there that the radar back-scattering parameter $\gamma(\theta_o)$ for a slightly rough surface, (where $\gamma(\theta_o)$= radar scattering cross-section per unit area of terrain divided by the cosine of the angle of incidence) is given by

(9) $\gamma(\theta_o) = 8 \cos^3 \theta_o \, T \, \overline{Z^2} \, K^4 \int \rho(r) \, J_o \, (2Kr \sin \theta_o) \, r \, d \, r$

where

$\overline{Z^2}$ = mean square surface roughness

$K = \dfrac{2\pi}{\lambda}$

λ = free-space wavelength

θ_o = angle of incidence

$\rho(r)$ = correlation function for the surface

and T is the term due to the effect of the complex dielectric constant of the surface. For the case of circular polarization, one can show that by substituting the results of Ref. 1 into Eqs. (5) and (6), the appropriate factors are

$$(10) \quad T_{rr} = \left| \frac{(\epsilon' - 1)}{2} \times \frac{\left[\sin^2 \theta_o + \cos \theta_o (\epsilon' - \sin^2 \theta_o)^{\frac{1}{2}}\right]^2 + \left[\epsilon' + (\epsilon' - 1) \sin^2 \theta_o\right]}{\left[\epsilon' \cos \theta_o + (\epsilon' - \sin^2 \theta_o)^{\frac{1}{2}}\right]^2} \right|^2$$

$$(11) \quad T_{r\ell} = \left| \frac{(\epsilon' - 1)}{2} \times \frac{\left[\sin^2 \theta_o + \cos \theta_o) \epsilon' - \sin^2 \theta_o)^{\frac{1}{2}}\right]^2 - \left[\epsilon' + (\epsilon' - \sin^2 \theta_o)^{\frac{1}{2}}\right]}{\left[\epsilon' \cos \theta_o + (\epsilon' - \sin^2 \theta_o)^{\frac{1}{2}}\right]^2} \right|^2$$

where

$\epsilon' = \epsilon_1 + j \, \epsilon_2$

ϵ_1 = relative dielectric constant of surface material

$\epsilon_2 = \dfrac{\sigma}{\omega \, \epsilon_o}$

σ = conductivity of surface material

ϵ_o = permittivity of free space.

These scattering coefficients show that the scattering from a slightly rough surface is the product of two factors, one of which involves only the surface structure and is independent of the material.

The other depends upon the dielectric constant of the material but is independent of the surface structure. The effect of T_{rr} and $T_{r\ell}$ upon the over-all radar return is shown in Figs. 2 and 3 for dielectric constants*of $\epsilon = 5 + j\,0.5$ and $\epsilon = 1.1 + j\,0.015$. For comparison, the terms T_{hh} and T_{vv} are also shown for $\epsilon = 5.5 + j\,0.5$, (See Ref. 1). When $\epsilon = 1.1 + j\,0.015$, by setting $\delta = \epsilon - 1$ and making the proper approximations the calculation of T_{rr} and $T_{r\ell}$ may be greatly simplified because of the small value of δ. The results are

$$T_{rr} \doteq \left| \frac{2\delta(1+\delta)+\delta^2 \sin^2 \theta_o}{\delta(1+\delta)\cos^2 \theta_o + 4\delta} \right|^2$$

$$T_{r\ell} \doteq \left| \frac{\delta^2 \sin^2 \theta_o}{\delta(1+\delta)\cos^2 \theta_o + 4\delta} \right|^2 .$$

These give good results when $\delta \ll 1$ except at extreme values of θ_o ($\theta_o \to 90^\circ$ or $\theta_o \to 0^\circ$).

II. SCATTERING FROM THE LUNAR SURFACE

With the possibility of manned space flight in the forseeable future, the structure of the lunar surface is becoming of great interest. In order to attempt to answer the question as to whether a soft landing on the moon with a space vehicle is possible, several groups have[2,3] been measuring the radar return from the moon. Their work has shown that the moon has a "quasi-smooth" surface, and that most of the return is simply a specular bounce off the portion of the moon nearest the earth. However, with high-power, short-pulse radar a definite pulse broadening is observed, indicating that there is return from the lunar surface at other than normal incidence. This part of the scattered field is the part predicted by the preceding theory, and it can give more information about the structure of the lunar surface than does the specular reflection. Because knowledge of the fine structure of the lunar surface is so completely lacking and the accuracy of the measured return from this portion of the lunar surface is poor, due to the small signal level (differences as large as 10 db exist between measurements by different observers[2,3]), no definite information about the structure can be given in this report. However, by applying the preceding work to the case of scattering by the lunar surface, certain interesting possibilities arise.

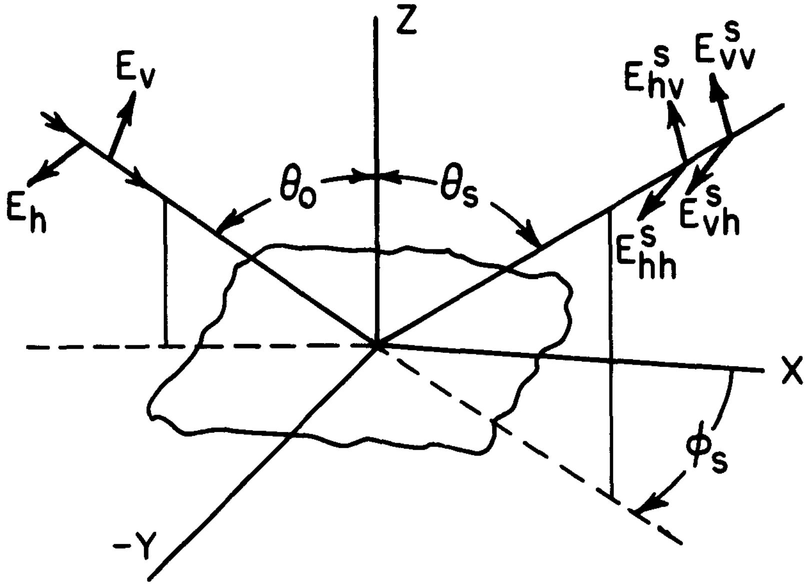

Fig. 1. Scattering from a slightly rough surface.

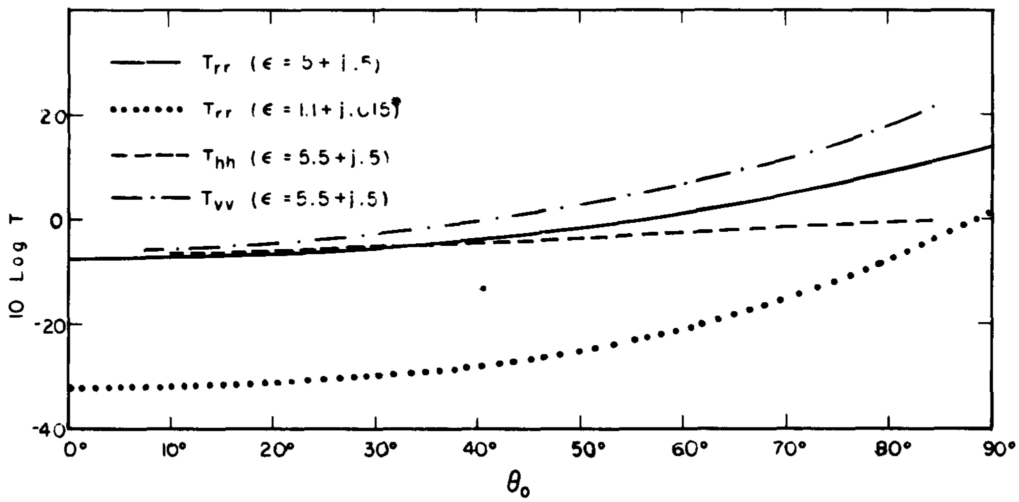

Fig. 2. Effect of the dielectric constant on return from a slightly rough surface.

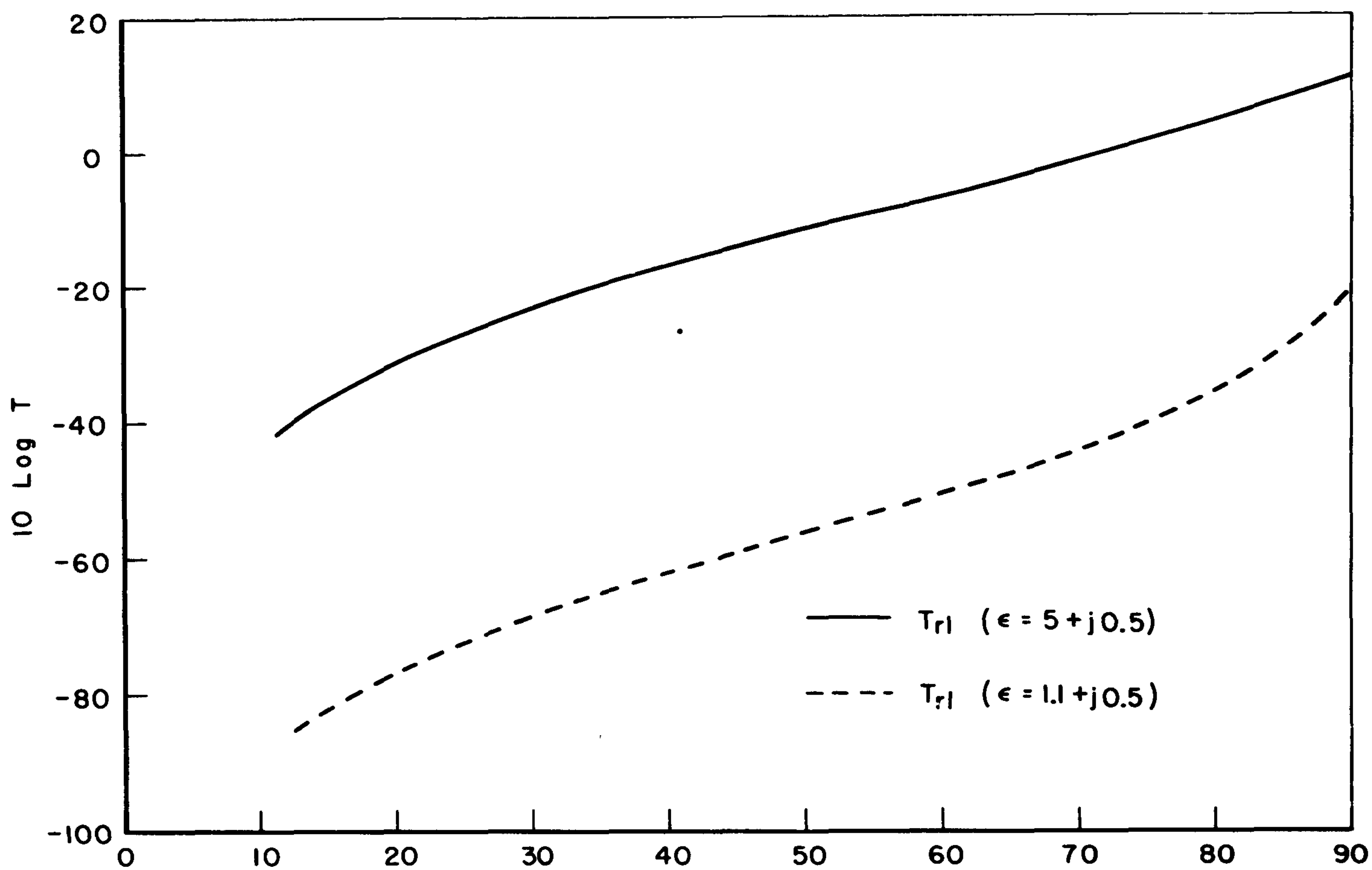

Fig. 3. Effect of the dielectric constant on return from a slightly rough surface (cross-polarized term).

The theoretical scattering coefficient, $\gamma(\theta_o)$, is shown in Fig. 4 for two different values of the parameters ϵ, $\rho(r)$, and $\overline{Z}^2$ along with the results measured by Pettengill.[2] In one case the complex dielectric constant is taken as $\epsilon = 1.1 + j\,0.015$, a value given by Senior and Siegel[4] for the dielectric constant of the moon at 400 mc. The other value, $\epsilon = 5 + j\,0.5$, was arbitrarily chosen. In each case the correlation function was taken to be of the form $\rho(r) = e^{-\frac{\pi r^2}{4\ell^2}}$. The values of $\overline{Z}^2$ and ℓ were the chosen to give close agreement with the measured results. In Fig. 5, the corresponding scattering coefficient for cross-polarization

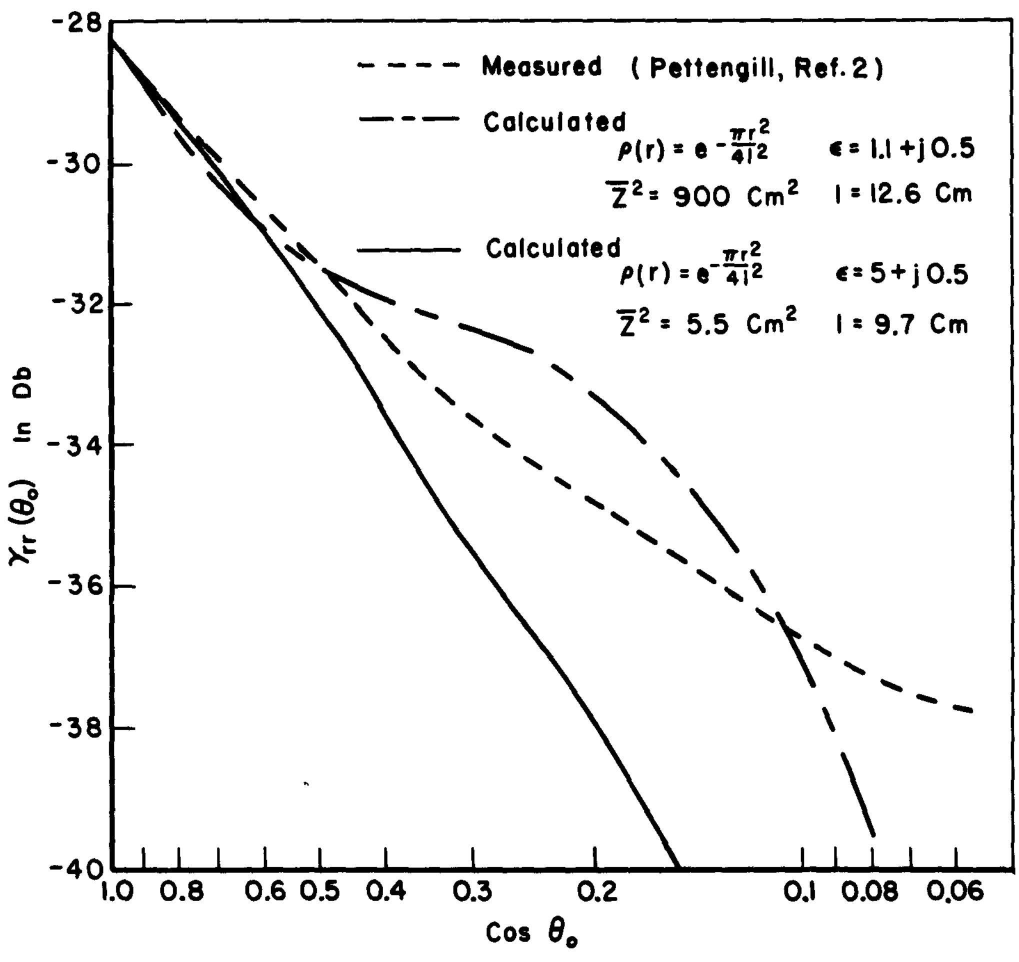

Fig. 4. Calculated and measured back scattering coefficient of the moon.

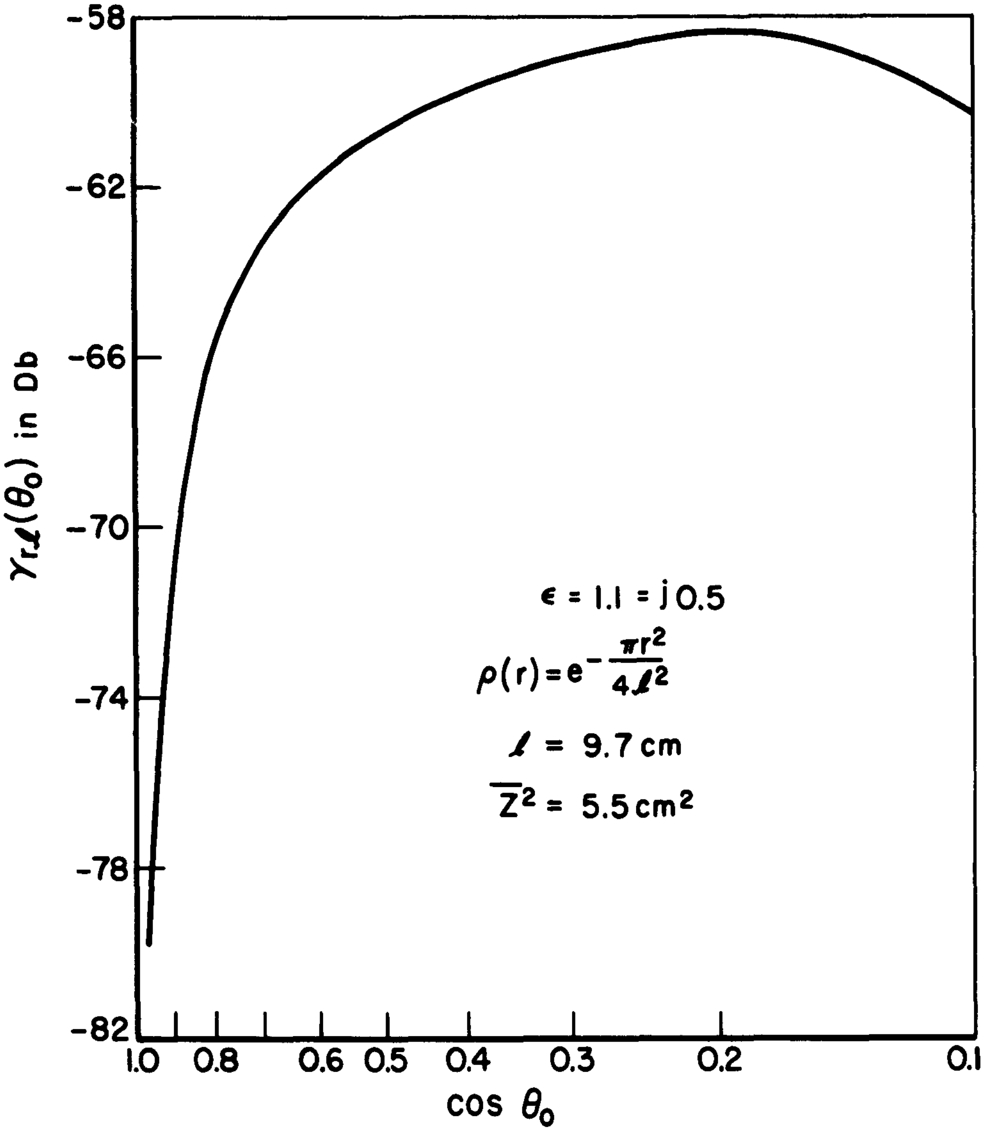

Fig. 5. Calculated and measured backscattering coefficient of the moon (cross-polarization).

and $\epsilon = 1.1 + j0.15$ is plotted as a function of $\cos \theta$.

Some interesting observations can be made at this point. Although fairly good agreement with the measured data is achieved by setting $\epsilon = 1.1 + j0.015$, the length scales ($\ell = 12.6$ cm, $(\overline{Z^2})^{1/2} = 30$ cm) required to produce agreement are such as to invalidate the theoretical assumptions on which Eqs. (9) -(12) are based. The most important of these assumptions was that the slope of the surface was everywhere small, i.e., the root-mean-square surface roughness $(\overline{Z^2})^{1/2}$ should certainly be considerably smaller than the mean scale ℓ.

It can also be seen by comparing Eqs. 2 and 3, or 4 and 5, that the cross-polarized return is lower than the direct return by 30 to 40 db for this type of surface model with $\epsilon = 1.1 + j0.015$. Choosing $\epsilon = 5 + j0.5$ produces a more realistic surface (ℓ=9.7cm)($(\overline{Z^2})^{1/2}$=2.3cm) ; however, for this case much poorer agreement with the angular dependence of measured values of the scattering coefficient results. Another interesting point is that the measurements indicate a definite limb brightening, at very small incidence angles. For $\epsilon = 1.1 + j0.015$ a limb brightening is also indicated; however, this occurs at slightly larger incidence angles. In order to produce the brightening at smaller angles an even smaller dielectric constant would be called for, and this would produce an even more unrealistic surface (larger $\overline{Z^2}$). When considering these results, several items must be kept in mind. First, the accuracy of the measurement is uncertain. Secondly, the preceding theory on scattering from a slightly rough surface may not meet the conditions existing on the lunar surface. Also, even if the Moon can be classified as a slightly rough surface, the correlation function and the complex dielectric constant of the surface material are both unknown. Consequently, conclusions drawn by comparing these theoretical results with the single measurement of Pettengill must be regarded as purely speculative at the present time.

III. FUTURE WORK

These results do, however, suggest several areas where experimental work could be done which might either substantiate or refute the slightly rough model of the lunar surface. The first and most obvious is the need for more reliable measurements of the scattering coefficient $\gamma_{rr}(\theta_o)$. Along these same lines a measurement of the reverse-polarized scattering coefficient $\gamma_{r\ell}(\theta_o)$ would be highly desirable. This measurement would not only help substantiate the theory, but it also might help determine the dielectric constant of the surface material. Figure 3 shows that the level of return for this reverse-polarized case is greatly dependent upon the dielectric constant of the scattering material.

A great number of measurements have been taken here [5] on surfaces described by the slightly rough model (asphalt, concrete, etc.); however, these were all made for a linearly polarized incident wave. It would be desirable to extend these measurements to the case of circular polarization to see if the theory gives comparable results. Also, since it is not likely that asphalt or concrete are representative of the lunar surface, measurements could be made for a large variety of surfaces, possibly including artificial models, in an attempt to find a surface that gives a return similar to that observed from the actual surface of the Moon.

IV. CONCLUSION

It has been shown that the "slightly-rough" model of the lunar surface can account for the observed radar cross-section of the Moon for angles not near normal incidence. However, no great reliance can be placed on this agreement because of the large uncertainities in the measurements and in the extent to which the slightly rough surface theory actually applies to the Moon. The results do show areas where more experimental work is desirable, including more reliable measurements on scattering by the Moon for both circular and reverse-circular polarization, and measurement of terrestrial surfaces, in an attempt to find one that has a scattering behavior similar to that of the lunar surface.

V. REFERENCES

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